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Risk-Return Profiles of Long-Term Saving Plans. Comparison of different saving strategies based on German capital market data 1955 - 2025

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Zusammenfassung

Auf der Grundlage historischer Kapitalmarktdaten für Deutschland (1955 - 2025) analysieren und vergleichen wir drei verschiedene Strategien für die Ansparphase langfristiger Sparprozesse:

- Constant Mix (CM)-Sparpläne mit einem konstanten Verhältnis von Aktienanlagen und Anlagen in sichere Anleihen,
- Life Cycle (LC)-Sparpläne; d.h. CM-Sparpläne in Verbindung mit einer Life Cycle-Strategie zum Ende des Sparprozesses,
- Collective Constant Mix (CCM)-Sparpläne.

Für diese drei Strategien der Ansparphase berechnen wir verschiedene Rendite-Risiko-Profile. Während die Rendite jeweils objektiv und genau gemessen werden kann, ist Risiko ein sehr komplexes Phänomen, das sich einer objektiven Definition und vor allem einer Messbarkeit weitgehend entzieht. Aus diesem Grunde müssen wir uns darauf beschränken, ausgewählte Aspekte des Risikos zu beschreiben und - soweit möglich - zu messen. Das zentrale Ergebnis unserer Untersuchung ist, dass CCM-Sparpläne signifikant bessere Rendite-Risiko-Profile aufweisen als CM- oder LC-Sparpläne.

Abstract

Using historical capital market data for Germany (1955-2025) we analyze and compare three saving strategies:

- constant mix (CM) saving plans,
- life cycle (LC) saving plans and
- collective constant mix (CCM) saving plans.

For these three saving strategies we calculate different risk-return profiles. While *return* can be measured objectively and precisely, *risk* is a very complex phenomenon that eludes an objective definition and, even more so, measurability. We therefore have to content ourselves with describing and, where possible, measuring only certain *aspects of risk*. Our main result is that *CCM*-saving plans have significant better risk-return profiles compared to *CM*- or *LC*- saving plans.

Risk-Return Profiles of Long-Term Saving Plans

Comparison of different saving strategies based on German capital market data 1955 - 2025

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Abstract

Using historical capital market data for Germany (1955-2025) we analyze and compare three saving strategies:

- constant mix (*CM*) saving plans,
- life cycle (*LC*) saving plans and
- collective constant mix (*CCM*) saving plans.

For these three saving strategies we calculate different risk-return profiles. While *return* can be measured objectively and precisely, *risk* is a very complex phenomenon that eludes an objective definition and, even more so, measurability. We therefore have to content ourselves with describing and, where possible, measuring only certain *aspects of risk*. Our main result is that *CCM*-saving plans have significant better risk-return profiles compared to *CM*- or *LC*-saving plans.

The outline of our paper is the following: After the introduction, in section 2 we describe the capital market data that are underlying our calculations. In section 3 we outline the technical details of *CM*-, *LC*- and *CCM*-saving plan; clearly, the *CCM*-model requires more definitions. We also define the risk measures underlying our risk-return profiles, which we present in section 4. We end with some concluding remarks.

1. Introduction

Pension systems worldwide consist of a combination of *Pay-as-You-Go* (*PAYG*) *systems* and *capital-funded* systems. From the macroeconomic viewpoint a *PAYG* system is linked to the *production factor labor* while a capital-funded system is linked to the *production factor capital*. Both systems have their merits and drawbacks. One argument in favor of capital funding is the fact that in most developed countries we observe an aging working population, that puts pressure on *PAYG*-Systems. However, a pension fund cannot directly invest into the production factor capital in the notion of national income accounting.[1] In particular an investment into corporate bonds only indirectly secures a share in capital stock of an economy since part of the profit generated by the company, namely a risk premium, goes to the shareholders. Similar considerations apply to government bonds. Thus - at least in theory - only a 100% real investment (stocks and real estate) would insure a full participation in the production factor capital. But a 100% equity investment means an unacceptable risk for most savers. So, capital funded pension systems are confronted with the *risk-return dilemma*: The

[1] Only a small fraction of the national capital (machinery, real estate, patents, intellectual property, …) are traded on capital (including real estate) markets.

money can be invested in risky asset (such as equities) with high returns or can be invested safely (e.g. in government bonds) with significant lower returns. The *equity risk premium* (*ERP*) is a well-established key figure to gauge the *average* extra return if one invests in equities rather than in bonds.[2] However, an individual saver who puts aside part of her/ his labor income during working life may face a stock market crash just at the moment she/ he wants to retire. The problem is that an individual saver (the same applies to an age cohort of savers) cannot realize the *average* return on equities.

Collective defined contribution (*CDC*) pension systems try to overcome this problem by some kind of collective agreement among generations of savers aiming to redistribute "deviations" from the average. [3] The idea of intergenerational risk transfer with respect to capital market risks is old since classical with-profit life insurance or endowment policies can be regarded as an implementation of some kind of intergenerational (capital market) risk transfer.[4] The last years research into intergenerational risk transfer arrangements has helped to better understand *CDC* pension schemes. Applying stochastic models (including stochastic simulation techniques) one can prove a positive welfare effect (e.g. Gordon/ Varian 1988, Gullién/ Jørgensen/ Nielsen 2006, Gollier 2008, Hoevenaars 2008, Westerhout 2011, Cui/ De Jong/ Ponds 2011, Goecke 2012, 2018) of *CDC*-arrangements or we can derive optimal ALM-strategies (Chen/ Kanagawa/ Zhang 2021, Kling/ Kramer/ Ruß 2025). However, to the best knowledge of the author, there are only few empirical studies with respect to *CDC* plans (e.g. Wesbroom/ Arends/ Turnock/ Harding 2015 with UK-Data). We want to fill this "empirical gap" by a backtesting based on German capital market data. Even though we are working with real world capital market data it should be clear, that the following is not a study of real *CDC*-pension systems; it is rather a study about how a *CDC*-pension system would have performed if a proper *CDC*-system had been installed decades ago.

The author is well aware of the fact that the following study only refers to the German market and that historic market data may not be representative for what might happen in future. However, one should keep in mind that even the most elaborated stochastic model is equally limited with respect to the predictive power – the future is only a realization of one of millions of possible future random walks![5]

The *CDC*-model underlying our backtesting is inspired by the continuous time (*c.t.*) model presented in Goecke (2013) and we want to check whether the theoretical results derived for

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[2] Cf. Jorda/ Knoll/ Kuvshinov/ Schularick/ Taylor (2019) for a comprehensive study of *ERP* worldwide; Damodaran (2025)
[3] Cf. Gollier (2008)
[4] Cf. Goecke (2003)
[5] We do not discuss the rather philosophical question whether stochastic models build on the mathematical concept of probability are in principle appropriate to model economic scenarios. The fundamental problem is that the mathematical concept of probability implicitly presumes that uncertain events can be repeated arbitrarily often - like throwing dices. Furthermore, it is obvious that economic scenarios are mainly result of human decisions partly driven by emotions and irrationalities.

the *c.t.* model remain valid for real capital markets. To this end firstly, we have to formalize a discrete time (*d.t.*) version and secondly, we have to select proper capital market data.

2. Capital Market Data and *CM*-, *LC*- and *CCM*-saving plans

2.1. Capital Market Data

We consider the time period from January 1955 to September 2025 and analyze all *n*-months savings plans within this time span. For each plan, we assume a fixed monthly contribution of one monetary unit, payable in advance. The savings contributions are invested in a portfolio comprising three asset classes:

- equities, represented by the *DAX* index,
- bonds, represented by the *REXP* index, and
- cash, represented by a money market index.

The *DAX* and *REXP* are well-established performance indices representing broadly diversified portfolios of German equities and German government bonds with varying maturities, respectively.[6]

The money market index employed in this study is based on the one-month money market rates (*EURIBOR*) published by the Deutsche Bundesbank. Since the published *DAX*, *REXP*, and *EURIBOR* data do not extend back to January 1955, it was necessary to reconstruct the historical index values on the basis of the available data.[7]

Let $t \in \{0, \ldots, 849\}$ denote the time index, where $t = 0$ corresponds to the Jan. 1, 1955 and $T = 849$ to Oct. 1, 2025. In our context, the beginning of each month is identified with the end of the foregoing month. *DAX*(*t*), *REXP*(*t*) and *Cash*(*t*) refer to the index value with respect to the end of month *t*. Note that by construction *Cash*(*t*) is known at time *t*-1, while *DAX*(*t*) and *REXP*(*t*) cannot be observed before time *t*.

To ensure comparability, most calculations are performed using inflation-adjusted values. Accordingly, we employ deflated (real) versions of *DAX*(*t*), *REXP*(*t*), and *Cash*(*t*). With respect to saving plans price adjustment means, that implicitly saving contributions are adjusted in line with the inflation rate.

For $\alpha, \beta \geq 0$ and $\alpha + \beta \leq 1$ we define

$$P_{(\alpha,\beta)}(0) = 100 \text{ and}$$

$$P_{(\alpha,\beta)}(t+1) = P_{(\alpha,\beta)}(t)\left(\alpha\,\frac{Cash(t+1)}{Cash(t)} + \beta\,\frac{DAX(t+1)}{DAX(t)} + (1-\alpha-\beta)\,\frac{REXP(t+1)}{REXP(t)}\right).$$

[6] Deutsche Börse AG, 2019 and Deutsche Börse AG, 2020

[7] For details see: https://www.th-koeln.de/mam/downloads/deutsch/hochschule/fakultaeten/wirtschafts_und_rechtswissenschaften/dokumentation_daten_en_20251113.pdf

$P_{(\alpha,\beta)}$ represents a portfolio with a constant mix of cash, equity and bond investments, subject to monthly rebalancing.

Important notice: All calculations are carried out on a net basis, i.e., without taking account of acquisition or administrative costs.

Constant Mix- and Life Cycle Saving Plans

We can now define a *CM*-saving plan starting at $t = t_0$ and maturing at $t = t_0 + n \leq T$:

$${}_{t_0}^{CM}S(0) \coloneqq 0,\ {}_{t_0}^{CM}S(k+1) \coloneqq \left(1 + {}_{t_0}^{CM}S(k)\right)\frac{P_{(\alpha,\beta)}(t_0+k+1)}{P_{(\alpha,\beta)}(t_0+k)} \text{ for } k = 0, 1, \dots n-1.$$

LC-saving plans start as a normal *CM*-saving plans, however towards maturity the proportion of risky assets is gradually reduce so that in the last month all money is invested in cash. Typically for *LC*-saving plans is a risk reduction in two staged: firstly, the equity share is substituted by bonds and at a second stage bonds and equities are substituted by cash. More formally: Given α, β and integers $k_\alpha \leq k_\beta \leq n$ we define

$$\alpha(j) := \begin{Bmatrix} \alpha & (j = 1, \dots, n - k_\alpha) \\ \alpha + (1-\alpha)\frac{j-n+k_\alpha}{k_\alpha} & (j = n - k_\alpha + 1, \dots, n) \end{Bmatrix},$$

$$\beta(j) \coloneqq \begin{Bmatrix} \beta & (j = 1, \dots, n - k_\beta) \\ \beta\left(1 - \frac{j-n+k_\beta}{k_\beta}\right) & (j = n - k_\beta + 1, \dots, n) \end{Bmatrix}.$$

It is straight forward how *LC*-saving plans $\left({}_{t_0}^{LC}S(k)\right)_{k=0}^{n}$ are calculated - we can omit the details. As a special case ($k_\alpha = k_\beta = 0$) we get *CM*-savings plan.

Collective Constant Mix Saving Plans

CCM-saving plans are based on a *collective saving model*. In the sequel we describe a simple model that is sufficient for our purposes.

The collective saving model for our backtesting is a discrete time adoption of the continuous time model presented in Goecke (2013). The discrete time interval is one month. We consider a pension fund with assets and liabilities – cf. Figure 1. The liability side of the balance sheet consists of two parts: the total of all individual accounts – denoted by $V(t)$ – and a capital reserve – denoted by $R(t)$; we have $P(t) = R(t) + V(t)$. We generally assume that $P(t) > 0$ and $V(t) > 0$; however, we allow for the possibility of underfunding i.e. $R(t) < 0$.

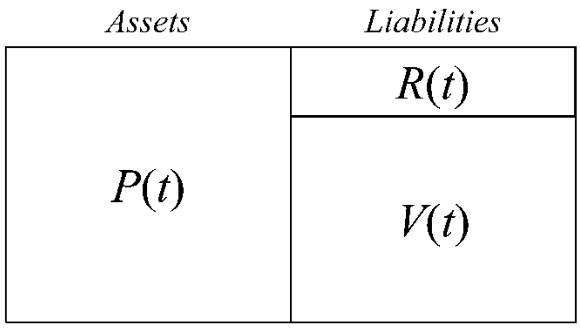

Figure 1: Balance sheet of a *CDC* pension fund

For simplicity we assume that the sum of contributions of all active savers just matches the total of money paid out to the retirees. By this assumption we rule out dynamic effects from a growing or shrinking population. In particular, we do not rebuild the start-up phase of our collective saving model. Instead we start with a balance sheet $(P(0), V(0), R(0))$ in $t = 0$ representing the initial situation on Jan. 1, 1955.

For $t = 0, \ldots, T$ we define the *reserve ratio* $\rho(t) := \ln\left(\frac{P(t)}{V(t)}\right)$. By varying the initial reserve ratio $\rho(0)$ we can simulate more or less favorable starting conditions.

In our collective savings model we do not actively manage the assets; instead we assume a portfolio with a constant equity ratio β and no investment in cash ($\alpha = 0$). Then we can write

$$P(t+1) = P(t)\exp\big(\mu_P(t+1)\big) \text{ with } \mu_P(t+1) := \ln\left(\frac{P_{(0,\beta)}(t+1)}{P_{(0,\beta)}(t)}\right).$$

The liabilities are actively managed: At the end of each month $[t, t+1]$ the *profit participation* $\eta(t+1)$ is determined, i.e. $V(t+1) = V(t)\exp\big(\eta(t+1)\big)$.

Since for every month the cash inflow equals cash outflow we get

$$\rho(t+1) = \rho(t) + \mu_P(t+1) - \eta(t+1) \qquad \textbf{(\textit{Eq 1})}$$

Since $\eta(t+1)$ is determined in view of $\mu_P(t+1)$ and $\rho(t)$, by (*Eq 1*) we see that the reserve ratio $\rho(t+1)$ can be fully controlled by some reasonable *liability rules*.

The following rules to determine the profit participation are intended to protect the savers from the volatility of the pension fund´s assets. This is done by allocations to or withdrawals from the collective reserve. However, in doing so one has to ensure that the reserve ratio is kept within reasonable bounds. To formalize, we need the following global parameters

- ρ_{min}, $\hat{\rho}$, ρ_{max} : *minimum*, *target* and *maximum reserve ratio*,
- $\theta \in [0, 1]$: *reserve adjustment parameter*,
- ERP: market *equity risk premium* for a 100% equity investment for one month.

The definition von $\eta(t+1)$ is done in three steps:

Step 1: $\eta_1(t+1) := \ln\left(\frac{Cash(t+1)}{Cash(t)}\right) + \beta\, ERP$

Step 2: $\eta_2(t+1) = \eta_1(t+1) + \theta(\rho(t) - \hat{\rho})$

Step 3: $\eta(t+1) = \min\{\eta_{max}(t+1), \max(\eta_2(t+1), \eta_{min}(t+1))\}$ with
$\eta_{min}(t+1) := \mu_P(t+1) + \rho(t) - \rho_{max}$ and
$\eta_{max}(t+1) := \mu_P(t+1) + \rho(t) - \rho_{min}$.

For price adjusted calculations we replace $\ln\left(\frac{Cash(t+1)}{Cash(t)}\right)$ by 0, i.e. $\eta_1(t+1) = \beta\ ERP$.

Remarks:

1. $\eta_1(t+1)$ is calculated at the beginning of month [*t*, *t*+1] and can be interpreted as the *expected return* of the pension assets.
2. If $\eta(t+1) = \eta_2(t+1)$ then
$$\rho(t+1) - \hat{\rho} = (1-\theta)(\rho(t) - \hat{\rho}) + \mu_P(t+1) - \ln\left(\frac{Cash(t+1)}{Cash(t)}\right) - \beta\ ERP.$$
If the *actual return* $\mu_P(t+1)$ equals the *expected return* $\beta\ ERP + \ln\left(\frac{Cash(t+1)}{Cash(t)}\right)$ the *reserve gap* $\rho(t) - \hat{\rho}$ is reduced by factor $(1-\theta)$. Thus Step 2 ensures that "in expectation" the $\rho(t)$-process is mean reverting provided $\theta > 0$.
3. $\eta(t+1) = \eta_{min}(t+1)$ implies $\rho(t+1) = \rho_{max}$ and $\eta(t+1) = \eta_{max}(t+1)$ implies $\rho(t+1) = \rho_{min}$.

We illustrate these rules for the time span Jan. 1955 to Jan. 1960 and for β = 100% - cf. Figure 2 and 3.

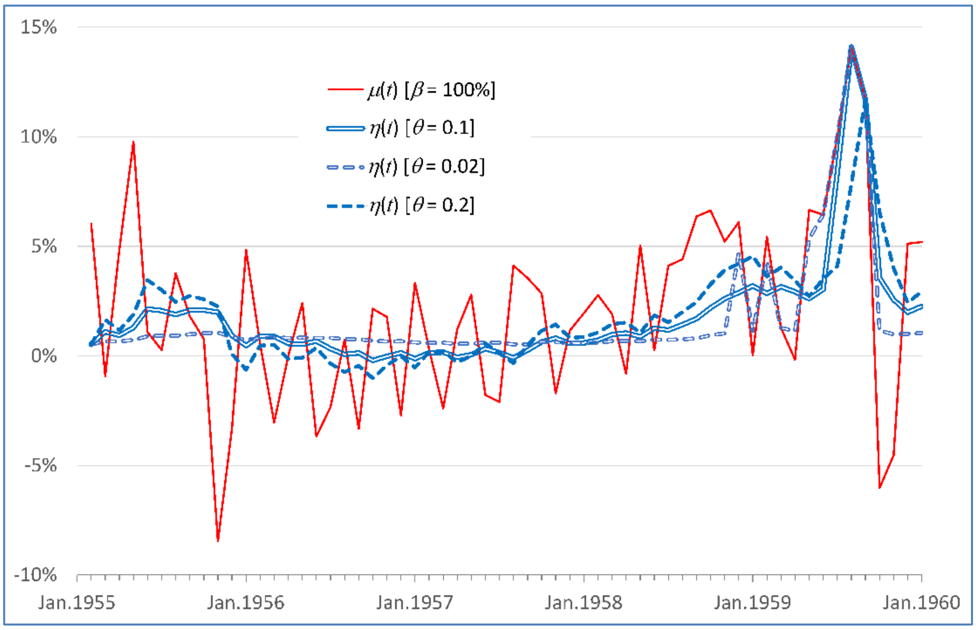

Figure 2: $\mu_P(t)$ und $\eta(t)$ for $\rho_{min} = 0$, $\rho_{max} = 0.5$, $\hat{\rho} = 0.2 = \rho(0)$, $\theta \in \{0.02, 0.1, 0.2\}$

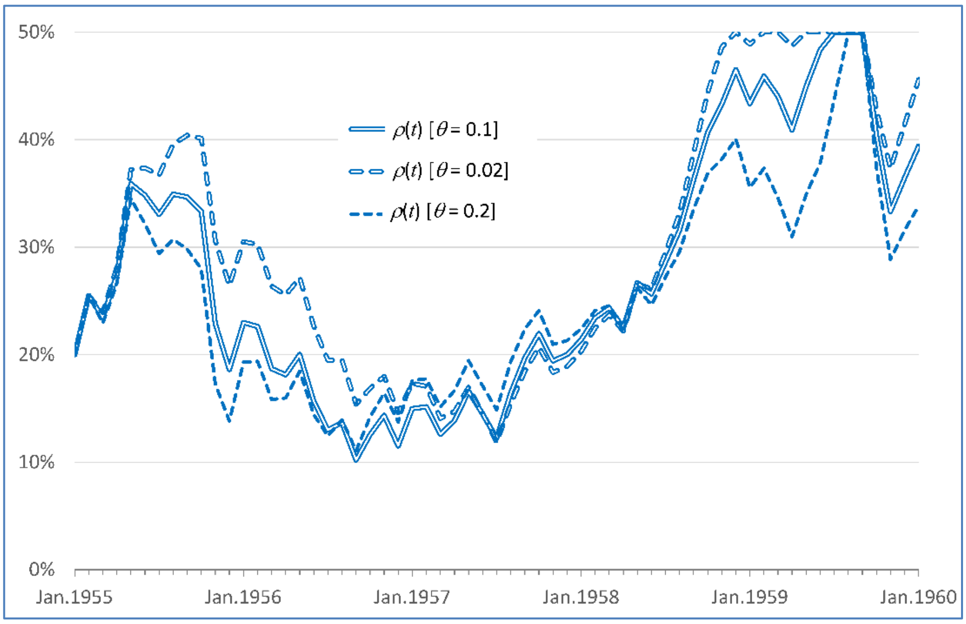

Figure 3: $\rho(t)$ for $\rho(0) = 0.2$, $\theta \in \{0.02, 0.1, 0.2\}$

We observe that θ controls the smoothing effect: Small θ-values result in a strong smoothing but only in combination with a more volatile reserve ratio $\rho(t)$ and a higher probability that $\rho(t)$ attains its limits ρ_{min} or ρ_{max} – see Figure 3.

Given the set of parameters (ERP, β, θ, ρ_{min}, ρ_{max}, $\hat{\rho}$) we can calculate CCM-saving plans $\left({}_{t_0}^{CCM}S(k)\right)_{k=0}^{n}$ on the basis of $\left(\eta(t_0+k)\right)_{k=1}^{n}$ for all $t_0 = 0, 1, \ldots, T\text{-}n$.

3. Risk Measures for Long Term Saving Plans

We do not enter into a discussion how to measure the risk of long-term saving plan. We just present different (statistical) risk measures which are *plausible* and mirror at least aspects of the risk. One should keep in mind that *risk* all statistical methods to measure have only limited explanatory power. The limits of measurability become particularly evident when one considers that *risk* also has a strong subjective component; the very same situation may be perceived as more or less risky by different individuals. We will return to this point again in our conclusion.

Let n denote the saving period in months, then there are $T\text{-}n\text{+}1$ successive generations of savers who start saving at $t_0 \in \{0, 1, \ldots, T\text{-}n\}$. For each *generation* t_0 and saving plan X ($X = CM$, LC or CCM) we use the following notations:

$\{{}_{t_0}^{X}\mu(1), {}_{t_0}^{X}\mu(2), \ldots, {}_{t_0}^{X}\mu(n)\}$: log- return rates during saving period:

for $X = CM$ this is a subsequence of $\left\{\log\left(\frac{P_{(\alpha,\beta)}(k+1)}{P_{(\alpha,\beta)}(k)}\right): k = 0,1,\ldots,T-1\right\}$;

for $X = CCM$ a subsequence of $\{\eta(k+1): k = 0,1,\ldots,T-1\}$;

for $X = LC$ the asset mix (α, β) is adjusted during the last n - k_β months.

Then ${}_{t_0}^{X}S(0) = 0$ and ${}_{t_0}^{X}S(k+1) \coloneqq \left(1 + {}_{t_0}^{X}S(k)\right) \exp\left({}_{t_0}^{X}\mu(k+1)\right)$ for $k = 0, 1, \dots n-1$.

${}^{X}r(t_0)$: rate of return as annualized interest rate, i.e. ${}_{t_0}^{X}S(n) = \frac{\left(1+{}^{X}r(t_0)\right)^{n/12}-1}{1-\left(1+{}^{X}r(t_0)\right)^{-1/12}}$;

${}^{X}\bar{r} \coloneqq \frac{1}{T-n+1}\sum_{t=0}^{T-n} {}^{X}r(t)$: average total return for all savings plans of type *X*.

We can now proceed defining five risk measures for saving plan of type $X \in \{CM, LC, CCM\}$.

Volatility *(Vola)*

${}^{X}Vola(t_0) \coloneqq \sqrt{12}\, StdDev\left\{{}_{t_0}^{X}\mu(k): k = 1, \dots, n\right\}$ for $t_0 = 0, 1, \dots, T-n$,

${}^{X}\overline{Vola} := Mean\left\{{}^{X}Vola(t_0): t_0 = 0, \dots, T-n\right\}$.

The annualized volatility ${}^{X}Vola(t_0)$ measures "stress" generation t_0 experienced during the saving period. If for example ${}^{X}Vola(t_0) = 0$ then generation t_0 experienced a constant interest rate throughout the saving period. Volatility is the statistical standard measure for the investment risk and is one of the key parameters for stochastic simulation models. However, this risk measure is very abstract and can hardly be made understandable to a layperson.

Maximum Drawdown (*maxDD*)[8]

${}^{X}maxDD(t_0) \coloneqq Max\left\{\frac{{}_{t_0}^{X}S(k)-{}_{t_0}^{X}S(l)}{{}_{t_0}^{X}S(k)} : 1 \le k \le l \le n\right\}$ for $t_0 = 0, \dots, T-n$,

${}^{X}\overline{maxDD} \coloneqq Mean\left\{{}^{X}maxDD(t_0) :\ t_0 = 0, \dots, T-n\right\}$.

${}^{X}maxDD(t_0)$ is the maximum relative loss in value during the saving phase of generation t_0. ${}^{X}maxDD(t_0) = 0$ means that for this generation the accrued wealth has increased from month to month. Note that ${}^{X}maxDD(t_0) = 0$ may be accompanied by a negative total return.

Maximum Loss Duration *(maxLD)*

${}^{X}maxLD(t_0) \coloneqq Max\left\{l-k:\ {}_{t_0}^{X}S(l) \le {}_{t_0}^{X}S(k) : 1 \le k \le l \le n\right\}$ for $t_0 = 0, \dots, T-n$,

${}^{X}\overline{maxLD} \coloneqq Mean\left\{{}^{X}maxLD(t_0) :\ t_0 = 0, \dots, T-n\right\}$.

${}^{X}maxLD(t_0)$ is the maximum time span (in months) during the saving phase of generation t_0 in which the accrued saving capital did not increase - despite regular contributions! If ${}^{X}maxLD(t_0) = 0$ then from month to month the accrued saving capital increase; but this does not imply that the saving efforts of generations t_0 were successful.

It is obvious that the risk measures *maxDD* and *maxLD* are closely related, they both are indicators of the frustration that savers may experience during the saving phase. However, one should note that a sharp drawdown of the accrued capital or a long time span with no

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[8] Cf. Goldberg, Mahmoud, O. (2017)

increase may be offset by a strong increase in the following months. Both risk measures can be seen as good indicators of the risk that the savings contract is terminated prematurely. Note that ${}^{X}maxLD(t_0) = 0$ is equivalent to ${}^{X}maxDD(t_0) = 0$.

Intergenerational Imbalance ***(IGI)***

$$ {}^{X}IGI(t_0) := StdDev\{{}^{X}r(t_0 - k): k = 0,1,\dots,11\} \text{ for } t_0 = 11,\dots,T-n, $$

$$ {}^{X}\overline{IGI} := Mean\{{}^{X}IGI(t_0): t_0 = 11,\dots,T-n\}. $$

${}^{X}IGI(t_0)$ measures the local stability of the total return of mature saving plans. A low value for ${}^{X}IGI(t_0)$ indicates that the total return of generation t_0 does not differ to much form the total return of the 11 preceding generations. Keeping in mind that from the macroeconomic point of view a capital funded pensions system should provide a fair participation in *capital* as a *factor of production*, it is obvious that with 12 months the total return of long-term saving plans should not differ too much.

Disappointed Expectation ***(DE)***

$$ {}^{X}DE(t_0) := \begin{cases} 1 & if\ {}^{X}_{t_0}S(n) < 60 + {}^{X}_{t_0}S(n-60) \\ 0 & if\ {}^{X}_{t_0}S(n) \geq 60 + {}^{X}_{t_0}S(n-60) \end{cases} \text{ for } t_0 = 11,\dots,T-n, $$

$$ {}^{X}DE := \frac{1}{T-n+1}\sum_{t_0=0}^{T-n} {}^{X}DE(t_0). $$

$60 + {}^{X}_{t_0}S(n-60)$ can be interpreted as the (conservatively) *estimated final wealth* for generation t_0 five years (60 months) before maturity. ${}^{X}DE$ is then the probability that under the saving regime X the final wealth is smaller than expected.

4. Results

4.1. Overview

The basic portfolios for our backtesting calculations will be a mixture of *DAX*- and *REXP*-investment with no investment into cash ($\alpha = 0$); thus, we write P_β instead of $P_{(\alpha,\beta)}$. The α-component is only relevant for *LC*-saving plans; for these we use the following standard parameters: $k_\alpha = 12$ and $k_\beta = 60$.

For *CCM*-saving plans we take $\rho_{min} = 0$, $\rho_{max} = 0.5$, $\hat{\rho} = 0.2 = \rho(0)$, $\theta = 0.1$ as standard parameters. Variations of these standard parameters will be discussed in section 4.3.

We first illustrate the performance of the basic portfolios for $\beta = 0\%, 10\%, \dots, 100\%$ with initial value $P_{(\beta)}(t_0) = 100$ – cf. Figure 4. We see that on average the performance of the basic portfolio is better for bigger values of β. But Figure 4 also illustrates the risk of a pure

DAX-investment: There are long periods with no real return on *DAX*-investment and heavy losses.[9]

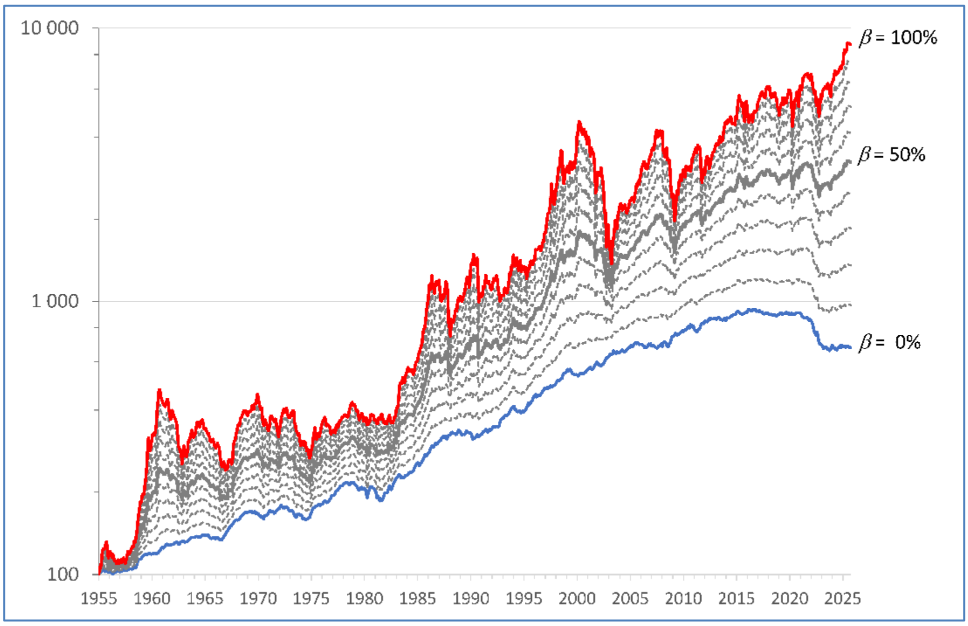

Figure 4: $P_\beta(t)$ for $t = 0$ (Jan.1955) to $t = T = 849$ (Oct.2025), price-adjusted, log-scale

Figure 5 shows the standard risk-return profile for $P_\beta(t)$ for β = 0%, 10%, …, 100% where *risk* is measured as annualized standard deviation of monthly log-return and *return* is defined as $\left(P_\beta(T)/P_\beta(0)\right)^{\frac{12}{T}} - 1$. As one would expect, the annualized volatility increases progressively for every 10% step-up of β. It is remarkable that the price adjusted profile is approximately a 2.7% left shift of the nominal profile independent of the equity ratio.

[9] There is no increase of *DAX* (price adjusted) between Oct. 1960 and April 1983; between March 2000 and April 2003 *DAX* (price adjusted) slumped by 69.9%.

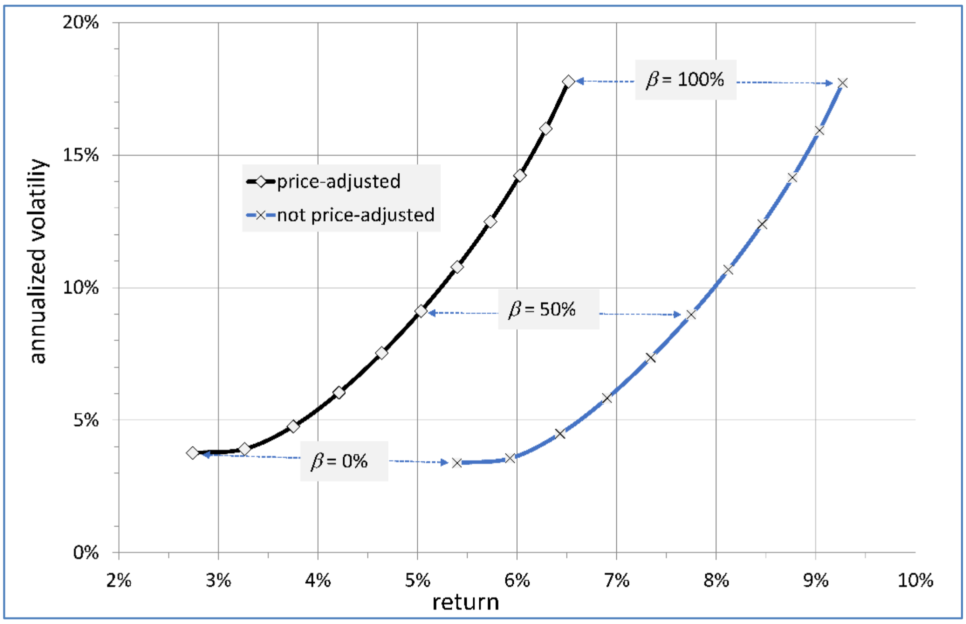

Figure 5: Risk-return profile of P_β - portfolios for β = 0%, 10%, …, 100%

Within the backtesting period we can evaluate 490 30-years saving plans and 370 40-years saving plans. If we only look at the total return ${}^{X}_{\square}r(t_0)$ for each generation t_0 we see remarkable difference between the generations of savers - despite price adjustment!

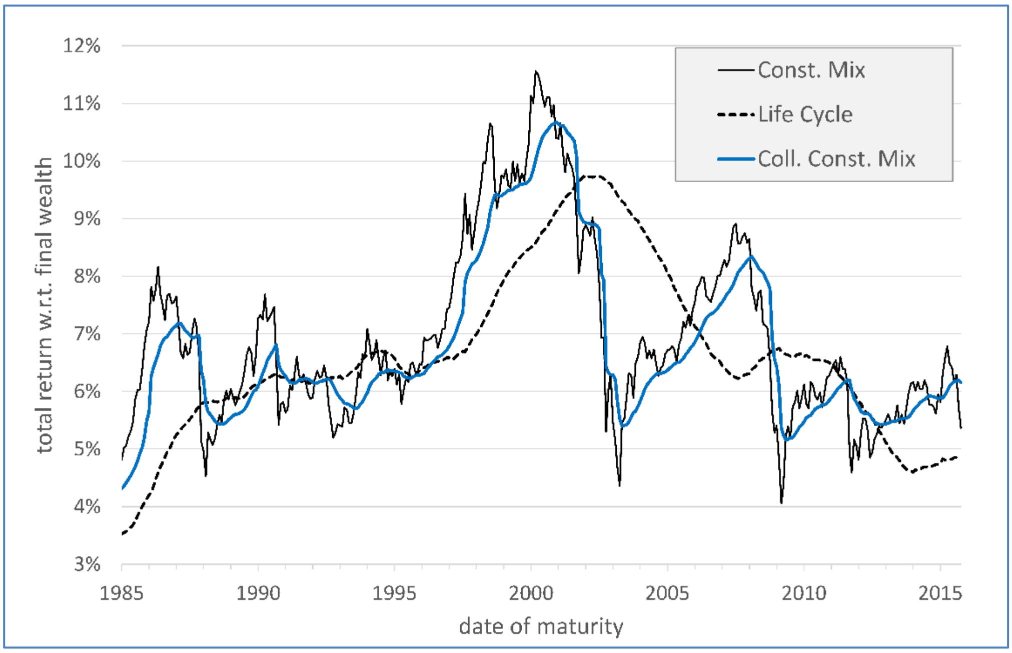

Figure 6: *Total return* (price adjusted) of 30-year saving plan for β = 100%

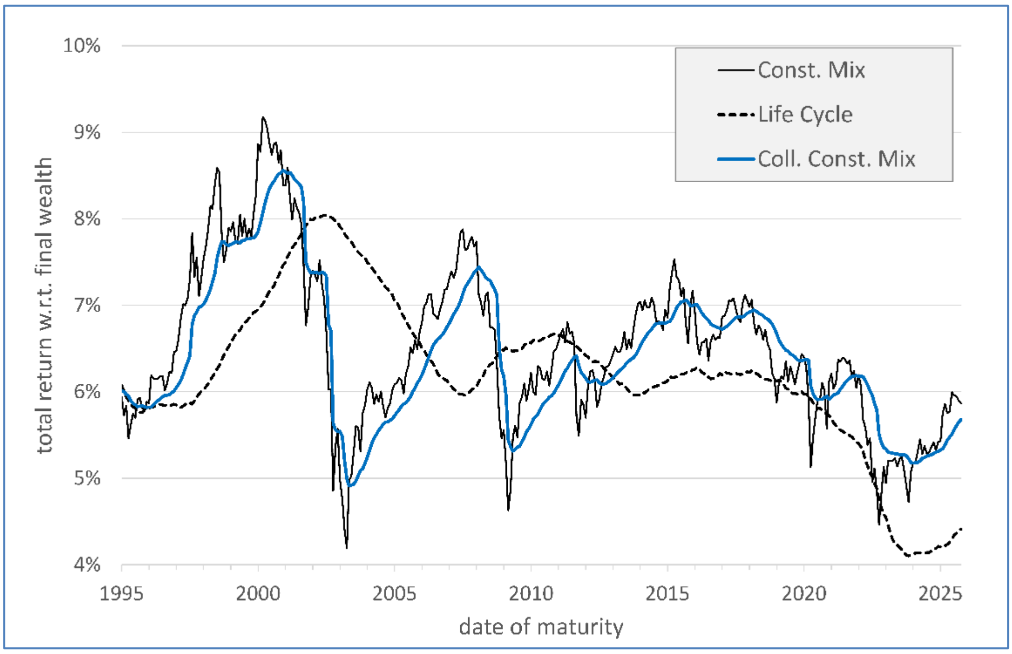

Figure 7: *Total return* (price adjusted) of 40-year saving plan for β = 100%

Figure 6 and 7 illustrate the characteristics of the different types of saving plans: The standard *CM*-plan follows very closely equity market trends which results in pronounced fluctuations of the total return. The total return function for *LC*-plans is much smoother: Due to the cooling off period during the last 5 years the performance becomes more and more decoupled from market developments. But the smoothness comes at the cost of lower returns – see Table 1 below. *CCM*-savings plans also produce a smoother total return function than standard *CM*-saving plans. However, *CCM*-savings plans are closer to the market since the underlying portfolio is always fully invested in equities. In contrast to *LC*-plans the smoothing is "free of charge" since mean and median return of *CCM*-plans are only slightly smaller than for *CM*-plans. This small difference only reflects the fact in our *CCM*-model the backtesting ends up with a reserve ratio over and above the initial reserve ratio.[10]

	Total Return ($^{X}r(t)$) [β = 100%, *price adjusted*]					
	saving period: 360 months (evaluation of 490 saving plans)			saving period: 480 months (evaluation of 370 saving plans)		
	CM	*LC*	*CCM*	*CM*	*LC*	*CCM*
Maximum	11.56%	9.73%	10.67%	9.17%	8.04%	8.56%
90%-Quantile	8.89%	8.62%	8.83%	7.80%	7.47%	7.66%
Median	6.18%	6.19%	6.10%	6.43%	6.19%	6.27%
Mean	**6.54%**	**6.07%**	**6.42%**	**6.52%**	**6.20%**	**6.42%**
10%-Quantile	4.96%	4.07%	5.07%	5.32%	4.87%	5.33%
Minimum	3.37%	3.03%	4.32%	4.19%	4.10%	4.91%

Table 1: Total return: 30- and 40-year savings plans – cf. Figure 6/ 7

[10] We start with $\rho(0) = 0.2$ and end with $\rho(T) = 0.29$.

4.2. Risk-Return Profiles

After this overview, we now evaluate for each single generation the risk indicators presented above and put them in relation to the *total return* ${}^{X}r(.)$.

In the following diagrams the x-axis always represents the (*average*) *total return* of 30- or 40-year savings plans with the underlying portfolio P_β for $\beta = 0\%, 10\%, \ldots, 100\%$. The y-axis represents one of the risk indicators ${}^{X}\overline{Vola}$, ${}^{X}\overline{maxDD}$, ${}^{X}\overline{maxLD}$, ${}^{X}\overline{IGI}$ and ${}^{X}DE$ for $X \in \{CM, LC, CCM\}$.

4.2.1. *Risk-Return Profile I*: Total Return vs. Volatility

We first use a classical risk measure namely the annualized volatility. However, the calculated volatility only refers to the saving period of the particular generation. For $n = 480$ (40 years) we get 390 data points for each $X \in \{CM, LC, CCM\}$.

For *CM*- and *LC*-plans the range of volatilities is roughly 14% to 21%; for *CCM*-plans we observe a much smaller rage (6% to 9%) – cf. Figure 8. It is worth noting, that

$$Correlation\left(\left({}^{X}r(t)\right)_{t=0}^{T-n}, \left({}^{X}Vola(t)\right)_{t=0}^{T-n}\right) = \begin{cases} -54.7\% & \text{for } X = CM \\ -64.8\% & \text{for } X = LC \\ -54.7\% & \text{for } X = CCM \end{cases}$$, which indicates that volatile markets during the saving period come along with lower returns.

In Table 2 we have summarized the data underlying Figure 8; we added the corresponding values for $n = 360$.

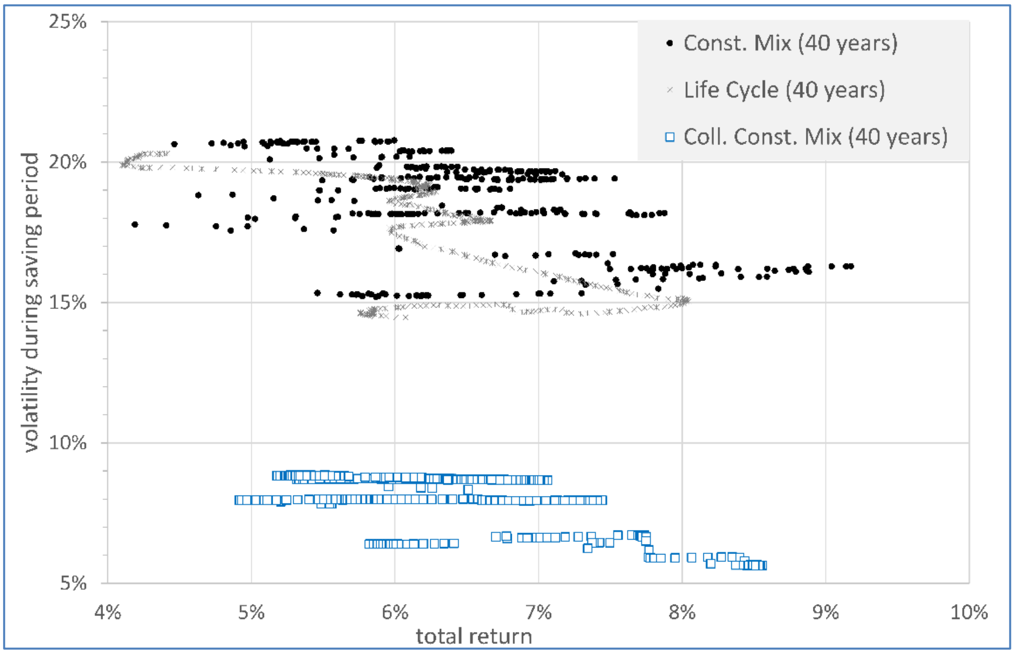

Figure 8: ${}^{X}r(t_0)$ vs. ${}^{X}Vola(t_0)$ ($t_0=0,\ldots,T-n$) for $n = 480$, $\beta = 100\%$ and $X \in \{CM, LC, CCM\}$

	Annualized Volatility (${}^{X}Vola$) [β = 100%, *price adjusted*]					
	saving period: 360 months (evaluation of 490 saving plans)			saving period: 480 months (evaluation of 370 saving plans)		
	CM	*LC*	*CCM*	*CM*	*LC*	*CCM*
Maximum	21.81%	21.03%	9.93%	20.77%	20.31%	8.86%
90%-Quantile	21.56%	20.57%	9.90%	20.59%	19.72%	8.79%
Median	19.68%	17.24%	8.92%	19.04%	17.90%	8.68%
Mean	**18.32%**	**16.94%**	**7.83%**	**18.49%**	**17.45%**	**7.97%**
10%-Quantile	14.56%	12.50%	5.13%	15.81%	14.69%	6.40%
Minimum	12.88%	12.22%	4.97%	15.20%	14.47%	5.64%

Table 2: Statistical evaluation of ${}^{X}Vola$ for 30- and 40-year savings plans – cf. Figure 8

To get the risk-return profile we have evaluate the volatility for all three saving plans X and for all basic portfolios P_β .

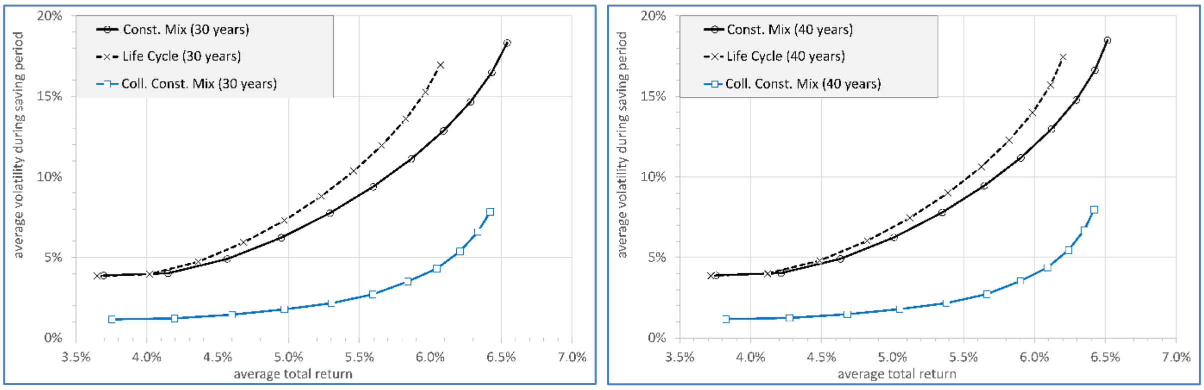

Figure 9 a/b: ${}^{X}\bar{r}$ vs. ${}^{X}\overline{Vola}$ for n = 360 and 480, β = 0%,…,100% and $X \in \{CM, LC, CCM\}$

Risk-return profile I of *CM*- and *CCM*-plans are more or less identical for n = 360 and n = 480 months. This is not true for *LC*-plans because the cooling down period (60 months) has more effect on the average return for n = 360 than for n = 480.

4.2.2. *Risk-Return Profile II*: Total Return vs. Maximum Drawdown

Figure 10 illustrates the risk-return profile of all 370 40-years saving plans. For *CM*- and *LC*-plans we observe two clusters of *maxDD* ≈ 39% and 68%. For *CCM*-plans we also have two clusters, however on lower levels *maxDD* ≈ 20% and 58%. These two clusters correspond to two local peaks of *DAX* on Oct. 1960 and March 2000.

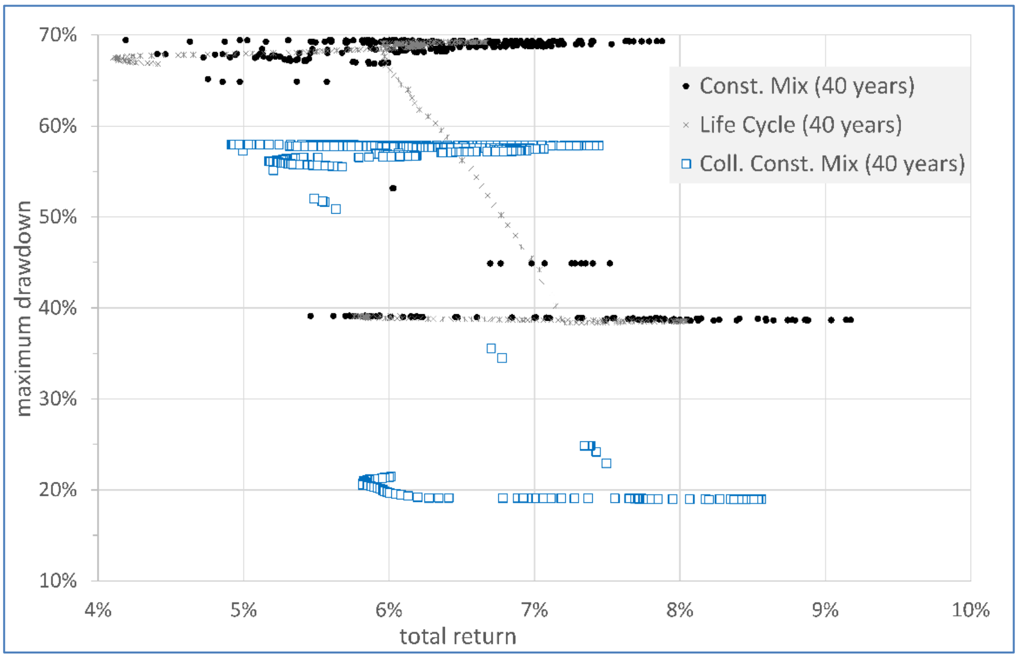

Figure 10: ${}^{X}r(t_0)$ vs. ${}^{X}maxDD(t_0)$ for $n = 480$, $\beta = 100\%$ and $X \in \{CM, LC, CCM\}$

The statistical evaluation of all 370 saving plans (see Table 3) gives only limited insight into the peculiar distribution of the *maxDD*-risk.

	Maximum Drawdown (${}^{X}maxDD$) [$\beta = 100\%$, *price adjusted*]					
	saving period: 360 months (evaluation of 490 saving plans)			saving period: 480 months (evaluation of 370 saving plans)		
	CM	*LC*	*CCM*	*CM*	*LC*	*CCM*
Maximum	69.10%	68.70%	57.64%	69.41%	69.28%	57.95%
90%-Quantile	68.79%	68.10%	57.35%	69.34%	69.19%	57.88%
Median	61.66%	47.93%	50.88%	68.65%	67.78%	57.22%
Mean	**53.90%**	**49.91%**	**39.44%**	**61.42%**	**57.98%**	**47.96%**
10%-Quantile	37.69%	30.47%	18.63%	38.79%	38.59%	19.05%
Minimum	28.64%	24.20%	17.62%	38.59%	38.36%	18.93%

Table 3: Maximum Drawdown: 30- and 40-year savings plans – cf. Figure 9

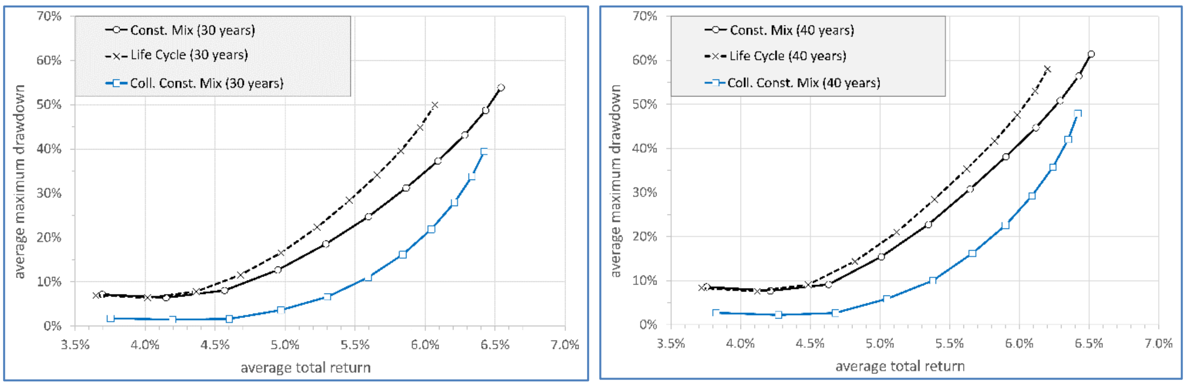

Figure 11 a/b: ${}^{X}\bar{r}$ vs. ${}^{X}\overline{maxDD}$ for β = 0%, …, 100% and $X \in \{CM, LC, CCM\}$

For equity ratios up to $\beta = 20\%$ the *maxDD*- risk does not increase for each of the three saving systems; for $\beta \geq 20\%$ the risk increases progressively. As for the volatility risk, the *CCM* saving plans have a better risk profile than *CM* and *LC*-plans. However, the superiority is not so pronounced as for the volatility risk. As we observed with respect to the volatility risk *LC*-plans underperform compared to *CM*- and *CCM*-plans.

4.2.3. *Risk Return Profile III*: Total Return vs. Maximum Loss Duration

We now analyze the *maxLD*-risk. Again, we start calculating the *maxLD*-risk all 40-year saving plans with $\beta = 100\%$ for the underlying portfolio – cf. Figure 12. For *CM*-plans we can make out three clusters at about *maxLD* ≈ 85/ 110 and 160 months.

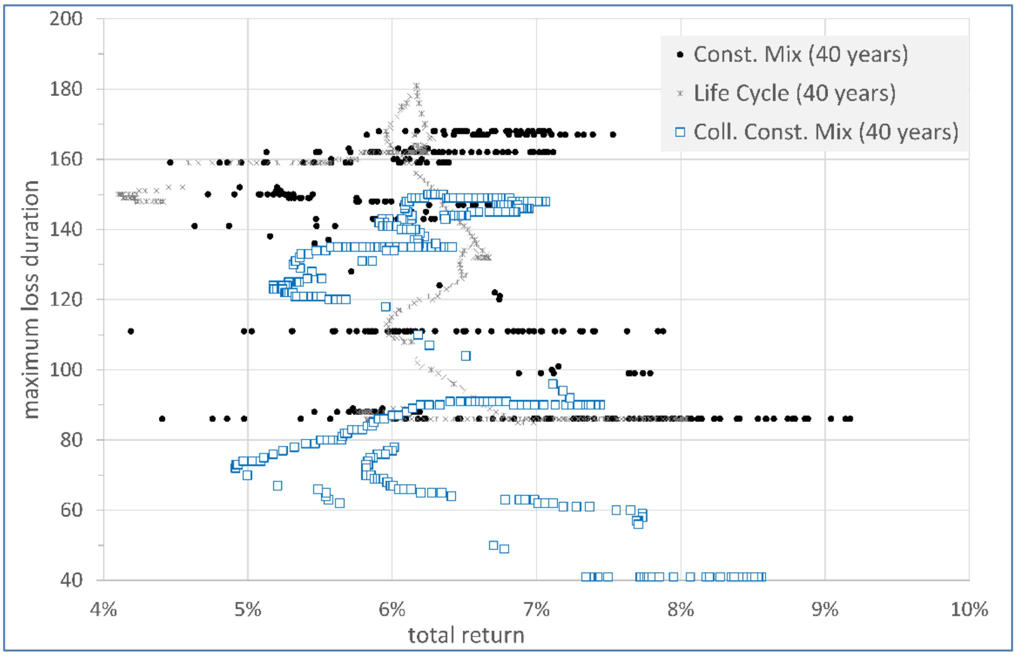

Figure 12: $^{X}r(t_0)$ vs. $^{X}maxLD(t_0)$ for $n = 480$, $\beta = 100\%$ and $X \in \{CM, LC, CCM\}$

	Maximum Loss Duration (^{X}mLD) [$\beta = 100\%$, *price adjusted*]					
	saving period: 360 months (evaluation of 490 saving plans)			saving period: 480 months (evaluation of 370 saving plans)		
	CM	*LC*	*CCM*	*CM*	*LC*	*CCM*
Maximum	159	163	135	168	181	150
90%-Quantile	148.00	147.00	125.00	167.00	164.00	148.00
Median	90.00	88.00	73.00	143.50	132.00	122.00
Mean	104.51	101.17	76.80	130.13	126.42	106.63
10%-Quantile	73.80	65.00	35.00	86.00	86.00	41.00
Minimum	61	57	31	86	85	41

Table 4: Maximum loss duration: 30- and 40-year savings plans – cf. Figure 10

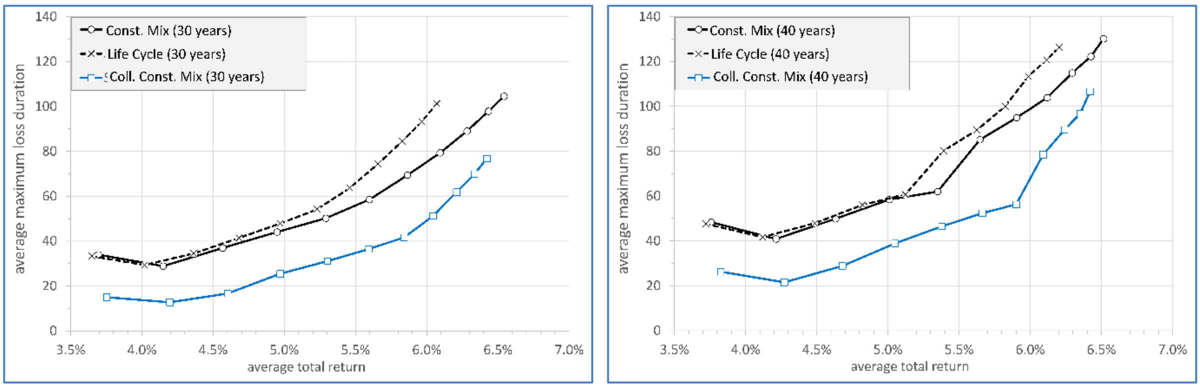

Figure 13 a/b: ${}^{X}\bar{r}$ vs. ${}^{X}\overline{maxLD}$ for $\beta = 0\%,\ldots,100\%$ and $X \in \{CM, LC, CCM\}$

Figures 13a/b very much resemble Figures 9a/b, indicating that the *mDD*- and *mLD* can be exchanged. However, we think it is worthwhile to consider both, the *mDD*- and the *mLD*-risk, since if a saver experiences a high drawdown or a long loss duration he or she presumably tends to cancel the saving process.

4.2.4. *Risk-Return Profile IV*: Total Return vs. Intergenerational Imbalance

We can regard ${}^{X}IGI(t_0)$ as in indicator of intergenerational fairness. If ${}^{X}IGI(t_0)$ is close to zero then the total return for generation t_0 does not differ to much from the total returns of the 11 proceeding generations. Purposely, we do not evaluate the volatility with respect to the total return of all generations because we think that *fundamental* capital market data do fluctuate. The consequence is that we have to accept that the result of a capital funded private pension plan is not stable but fluctuates. The same is obviously true for a pay as you go system, where the pension level is linked to the wage level.

As Figure 14 shows, for *LC*-saving plans ${}^{X}IGI(t_0)$-value cluster between 0% and 0.2% with only a few outliners above 0.2%. Similar clusters can be observed for *CCM*-plan but with clear outliners up to 0.9%.

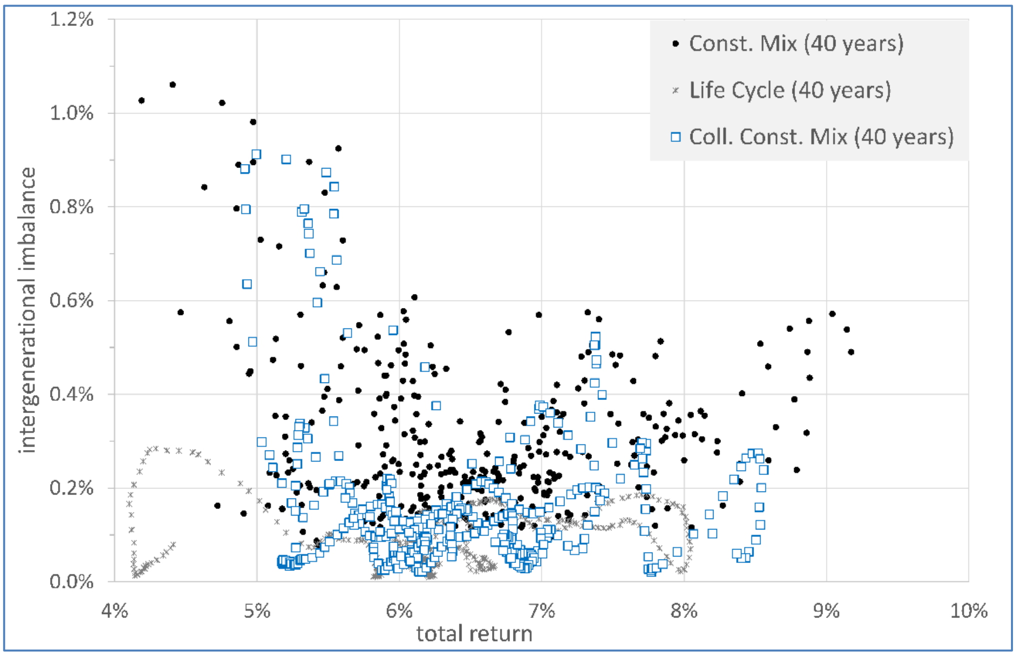

Figure 14: ${}^{X}r(t_0)$ vs. ${}^{X}IGI(t_0)$ for $n = 480$, $\beta = 100\%$ and $X \in \{CM, LC, CCM\}$

	Intergenerational Imbalance (${}^{X}IGI$) [$\beta = 100\%$, *price adjusted*]					
	saving period: 360 months (evaluation of 490 saving plans)			saving period: 480 months (evaluation of 370 saving plans)		
	CM	*LC*	*CCM*	*CM*	*LC*	*CCM*
Maximum	1.48%	0.34%	1.29%	1.06%	0.28%	0.91%
90%-Quantile	0.75%	0.25%	0.53%	0.51%	0.17%	0.34%
Median	0.37%	0.09%	0.15%	0.25%	0.07%	0.12%
Mean	0.43%	0.12%	0.23%	0.29%	0.08%	0.17%
10%-Quantile	0.19%	0.03%	0.05%	0.13%	0.02%	0.04%
Minimum	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

Table 5: Intergenerational imbalance: 30- and 40-year savings plans – cf. Figure 14

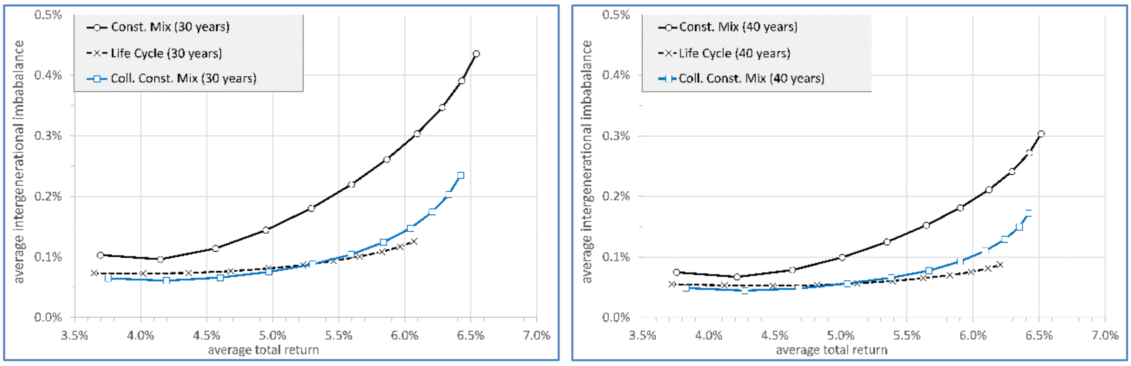

Figure 15a/b: ${}^{X}\bar{r}$ vs. ${}^{X}\overline{IGI}$ for $\beta = 0\%,\ldots,100\%$ and $X \in \{CM, LC, CCM\}$

With respect to the *IGI*-risk measure *LC*-plans outperform *CM*- and (partially) *CCM*-plans. This is what one would expect because of the 60-months cooling down period neighboring generations of saver experience correlated returns. But this advantage of *LC*-plans come along with the risk of missing an overperformance of equity market during the last 60 months.

4.2.5. *Risk-Return Profile V*: Total Return vs. Disappointed Expectation

Finally, we analyze the risk of disappointed expectation (*DE*). At a first glance one would expect that *LC*-plans outperform the others since the switching from volatile equity investment to bonds and cash should increase predictability. But actually, this is not the case – cf. Figure 16a/b. Quite the opposite is true: Compared to standard *CM*-plans not only the total return is lower but also the *DE*-risk higher. Here again *CCM*-plans outperform both *CM*- and *LC*-plans.

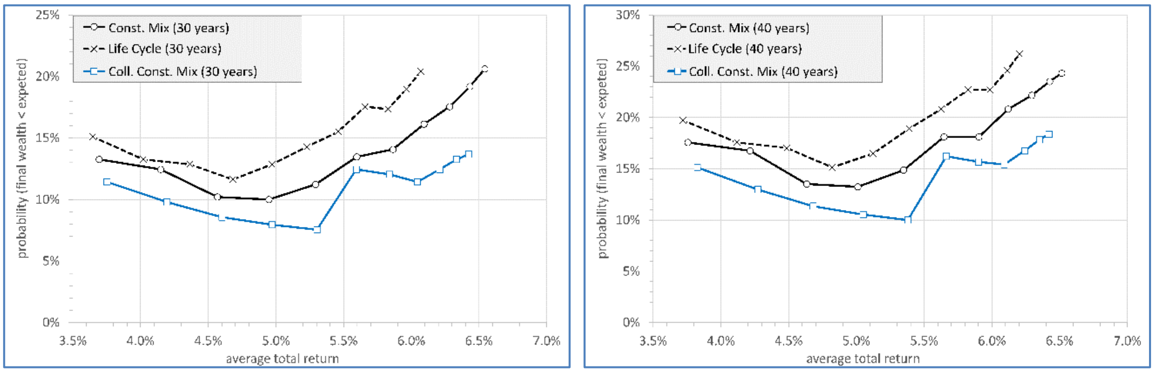

Figure 16a/b: ${}^{X}\bar{r}$ vs. ${}^{X}DE$ for $\beta = 0\%,\ldots,100\%$ and $X \in \{CM, LC, CCM\}$

4.3. Variation of *CCM*- Standard Parameters

All calculations are done on the basis of *price adjusted* capital market data

4.3.1. Variations of $\rho(0)$, ρ_{min} and ρ_{max}

We start with alternative values for $\rho(0)$, namely $\rho(0) = \rho_{min} = 0$ and $\rho(0) = \rho_{max} = 0.5$.

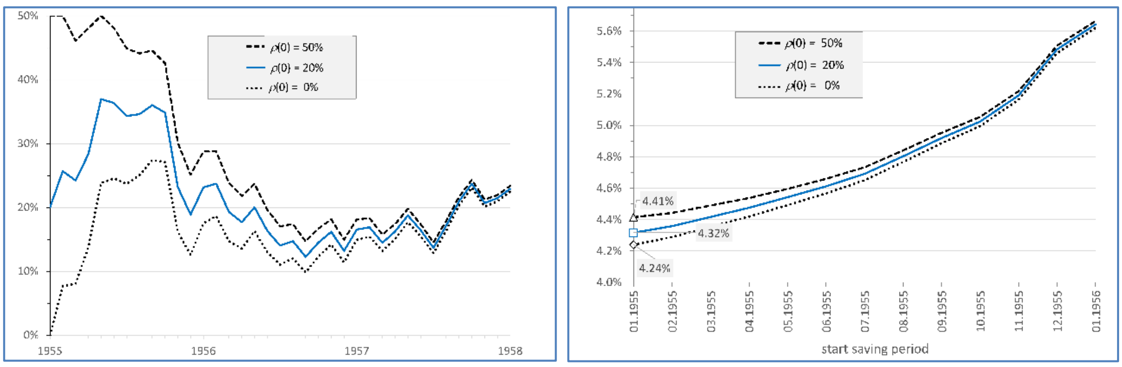

Figure 17a/b: $\left(\rho(t)\right)_{t=0}^{36}$ and $\left({}^{CCM}r(t)\right)_{t=0}^{12}$ for $\beta = 100\%$, $n = 360$ and $\rho(0) \in \{0, 0.2, 0.5\}$

Figure 17a shows the development of the $\rho(t)$- paths for the first three years: we see that after three years the differences between the alternative $\rho(t)$- paths is neglectable. With respect to the total return ${}^{CCM}r(t)$ the initial $\rho(0)$-value is very small even for the first generation $t = 0$. For $n = 480$ the ${}^{CCM}r(t)$ -paths for the three alternative initial reserve ratios are almost indistinguishable; in particular we observe ${}^{CCM}r(0)$ = 5.97% / 6.01% and 6.08% for $\rho(0) = 0$ / 0.2 and 0.5, resp. To sum up, we can conclude that the initial reserve ratio has only limited effect on the risk-return profile.

We now analyze the effect if we calculate without lower and upper boundary for the reserve ratio $\rho(t)$; i.e. we set $\rho_{min} = -\infty$ and $\rho_{max} = +\infty$.

In this case we have $\eta(t+1) = \eta_1(t+1) + \theta(\rho(t) - \hat{\rho})$ and thus the $\rho(t)$-path will not diverge to $-\infty$ or $+\infty$, provided $\theta > 0$. From Figure 8 we see that the "unlimited" $\rho(t)$-path actually goes beyond $\rho_{min} = 0$ and $\rho_{max} = +0.5$, but quickly returns to the standard $\rho(t)$-path.

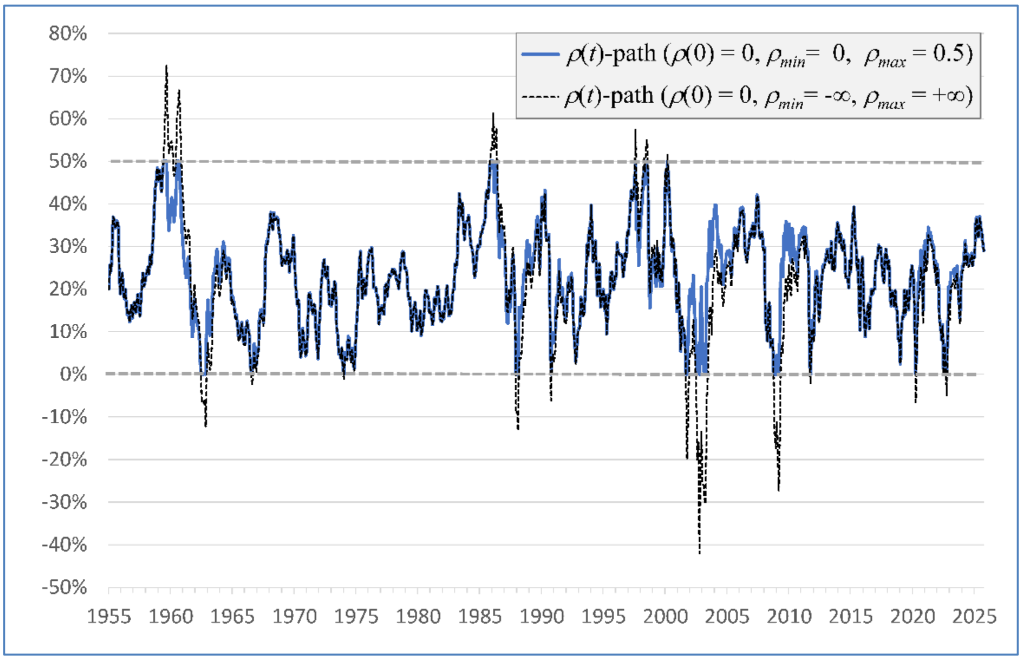

Figure 18: $\left(\rho(t)\right)_{t=0}^{T=849}$ for β = 100% and two different (ρ_{min} , ρ_{max})-settings[11]

It is obvious that in case there are no upper or lower bounds the profit participation $\eta(t)$ can be kept stable even if $\rho(t)$ approaches 0 or 0.5. As a consequence, the volatility risk measure ${}^{CCM}\overline{Vola}$ is reduce by lowering ρ_{min} or increasing ρ_{max}.[12] We do not go into details since a negative reserve means a burden for the following saver generations and an excessively high reserve ratio disadvantages age cohorts close to retirement.

[11] For $\beta \leq 50\%$ the relaxation of upper or lower bounds has no effect.
[12] As can be shown, the same applies to the other risk measures.

4.3.2. Variations of θ

The *reserve adjustment parameter* θ controls the speed by which the reserve ratio $\rho(t)$ is pushed towards the *target reserve ratio* $\hat{\rho}$. We evaluate the five risk measures for three alterative levels of the reserve adjustment parameter, namely $\theta = 0.02$, $\theta = 0.1$ and $\theta = 0.2$ – cf. Figure 19a-e.

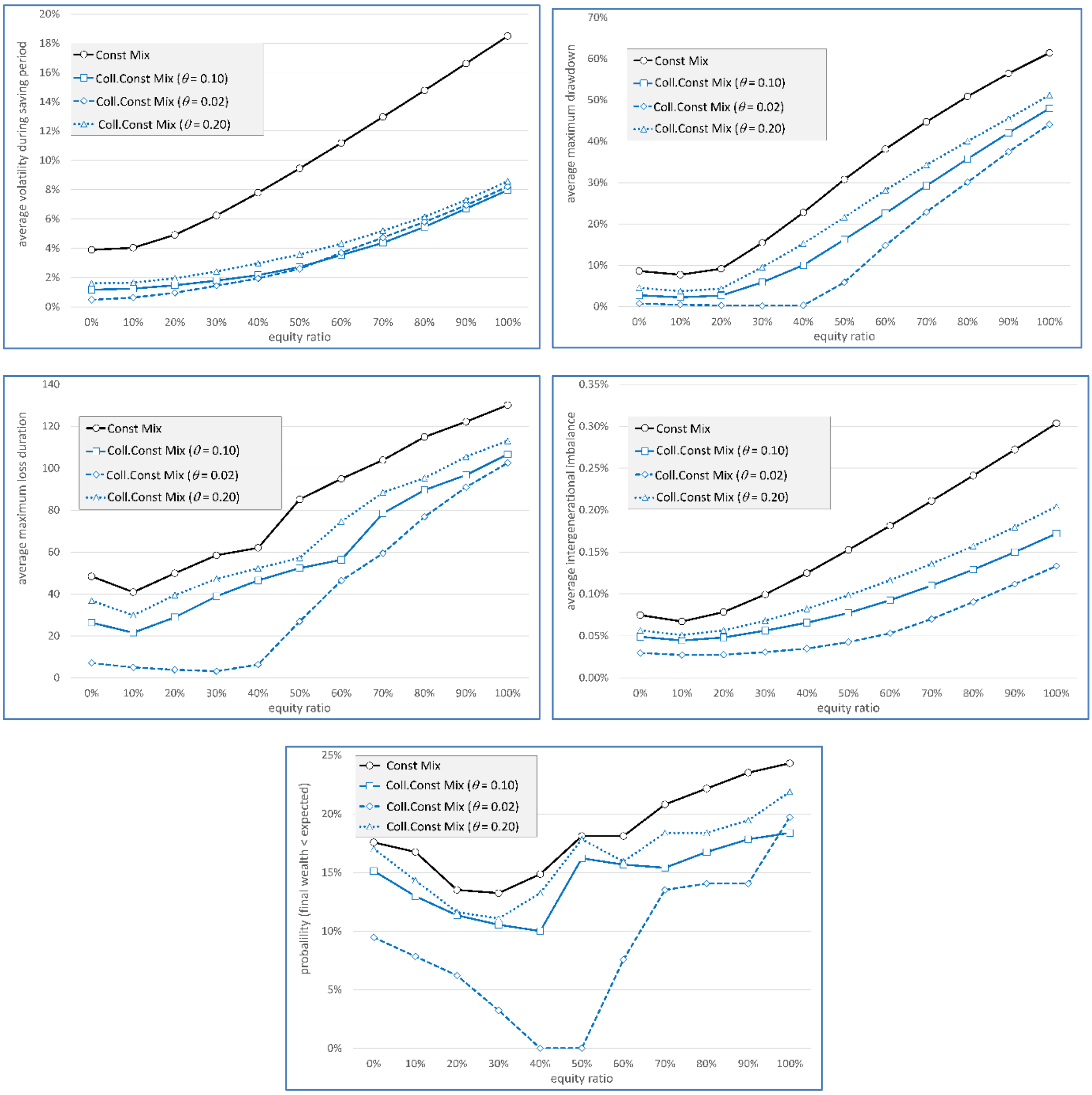

Figure 19a-e: $\{\overline{{}^{X}Vola}, \overline{{}^{X}maxDD}, \overline{{}^{X}maxLD}, \overline{{}^{X}IGI}, \overline{{}^{X}DE}\}$ for $\beta = 0\%,\ldots,100\%$, $n = 480$ and $X \in \{CM, CCM\ (\theta = 0.02, 0.10, 0.20)\}$

Except for $\overline{{}^{X}Vola}$, we observe a clear risk reduction for $\theta = 0.02$ compared to $\theta = 0.1$ or $\theta = 0.2$ for all risk measures. The risk reduction for $\theta = 0.02$ is particularly pronounced with respect to the *maxDD*-, *maxLD*- and *DE*-risk. With respect to $\overline{{}^{X}Vola}$ we observe that the variation of θ has only very limited effect.

It seems that a low θ-value is to be preferred, but a look at Fig. 20 reveals that for $\theta = 0.02$ the recovery of the total return is delayed, especially for the generations whose accrued capital matures between 2003 and 2008 and between 2010 and 2016. Also, we observe that on average the total return for $\theta = 0.02$ is lower that the total return for $\theta = 0.10$ and $\theta = 0.20$. The reason for this is that the average reserve ratio $\theta = 0.02$ is 26.7% compared to 23.0% (for $\theta = 0.10$) and 21.6% (for $\theta = 0.20$).

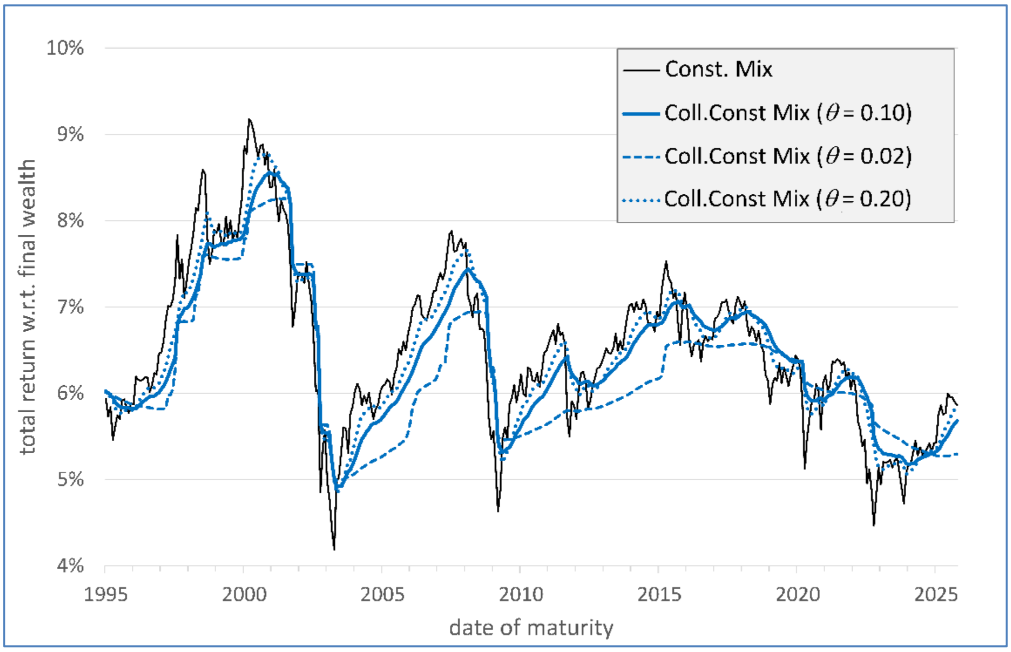

Figure 20: *Total return* ($\beta = 100\%$, $n = 480$) for *CM*- and *CCM*-plans ($\theta = 0.1, 0.02, 0.2$)

5. Concluding remarks

The question we asked was: Can investment risks w.r.t. long-term saving plans be reduced by introducing a collective reserve? The answer is a clear yes! This is not really surprising since *allocations to* and *withdrawals from* the collective reserve create a risk balance between saver generations. Provocatively, one could also say: The introduction of a collective reserve means that some generations of savers are expropriated in favor of others! But a look at Figure 20 reveals that the "normal" *constant mix* asset strategy results in enormous differences between closely successive generations of savers. This can't be explained by fundamental economic data: it is just a result of dramatic ups and downs in the equity market.[13] There is strong evidence that these extreme ups and downs are the result of *irrational exuberances*.[14] From this perspective we can regard collective saving plans as an insurance against irrational fluctuation on the equity markets. As any fair insurance contract an intergenerational risk transfer provides utility!

[13] Between March 2000 and April 2003, the *DAX* lost almost 70% of its price adjusted value.
[14] Cf. *Shiller* (2014)

We employed a very simple rule to manage the collective reserve. We did not discuss the question of how to manage the collective reserve in an optimal way because such an undertaking is clearly inadmissible if applied to one special time series as in our backtesting. But what we can do is to try to formulate some general rules:

- The principle of intergenerational justice demands for a minimal and maximal reserve ratio. It also makes sense to have a target reserve ratio.
- The fixing of the profit participation must be done by a neutral person or institution. But the question who this could be: An individual or a private institution can always very easily be suspected of pursuing their own interests. The same applies to a governmental institution. Independent public institutions, like regulatory authorities or central banks, might be suitable candidates.
- Saver generations close to maturity can no longer rely on the cost average effect. Different to our model, it is therefore worth considering to have a different profit participation rule for near maturity generations. This is then a kind of life cycle system without giving away the equity risk premium.
- We also refrained form an active asset management. Given the current reserve ratio and in view of the current economic outlook, it may be appropriate to adjust the equity allocation up or down. But this is not risk free since the asset manager might fall into the *irrational exuberance*- trap.

Risk also has a *social dimension*. Let us put ourselves in the position of a saver whose retirement income is already secured through a statutory or occupational pension scheme. For this person, additional private retirement provision is not strictly necessary and mainly serves to allow for a few extra comforts during retirement. He or she can therefore afford to invest in a riskier and, on average, more profitable asset.

The situation is entirely different for a saver whose additional private pension provision is the very means of ensuring a subsistence-level income in old age; such a person will tend to choose a safer and thus, on average, less profitable form of investment.

It can therefore be expected that the success of private retirement provision will be below average precisely among those who depend on this pillar of retirement income the most. To make matters worse, this group of people may be particularly susceptible to manipulation or distortion of their risk perception.

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